5. $48\frac{17}{29} \times 49\frac{12}{29} = 2400\frac{697}{841}$; $1295\frac{57}{93} \times 1296\frac{36}{93} = 1677615\frac{7343}{8649}$.

6. $7464 \times 7536 = 56248704$; $88044 \times 87956 = 7744000000 - 1936$.

7. $2777\frac{7}{9} \times 41666\frac{2}{3} \times 666\frac{2}{3} \times 54 \times 24 \times 52 \times 7692307\frac{9}{13} \times 625 \times 125 \times 56 \times 32 \times 142857142857142 8\frac{4}{7} \times 2083\frac{1}{3} \times 48 \times 833\frac{1}{3} \times 3125 \times 68543764287590 = \ldots$''

These and eleven other even longer computations are carried out mentally by Mr. Case.

The principle, $a^2 = (a-b)(a+b) + b^2$, may be used in the squaring of any number, though it is not so readily used if the numbers consist of more than two digits. Thus,

$$87^2 = (87-3)(87+3) + 3^2 = 84 \times 90 + 9,$$
$$92^2 = (92-2)(92+2) + 2^2 = 90 \times 94 + 4.$$

This is the principle used in several of Mr. Case's calculutions. Thus, $(5\frac{1}{2})^2 = (5\frac{1}{2} - \frac{1}{2})(5\frac{1}{2} + \frac{1}{2}) + (\frac{1}{2})^2 = 30\frac{1}{4}$.

ED. F.

330. Proposed by R. D. CARMICHAEL, Princeton, N. J.

An important function in the Theory of Numbers is one defined thus: $f(x)=1$ when $x>0$, $f(x)=0$ when $x=0$, $f(x)=-1$ when $x<0$. Two analytic expressions for $f(x)$ are the following:

$$f(x) = \lim_{n \doteq \infty} x^{1/(2n-1)},\ n=1, 2, \ldots;\quad f(x) = \lim_{n \doteq \infty} \frac{(x+1)^n - (x+1)^{-n}}{(x+1)^n + (x+1)^{-n}},\ x > -1.$$

It is required to find other non-trigonometric analytic expressions for this function. (There are several representations of $f(x)$ by means of trigonometric functions.)

Remark by the PROPOSER.

Professor F. H. Safford, of the University of Pennsylvania, has sent me the following expressions for the function defined in the problem:

$$\frac{2}{\pi}\int_0^\infty \frac{\sin xz}{z} dz,\quad \frac{2}{\pi}\int_0^\infty \frac{x\,dz}{x^2+z^2},\quad \lim_{m=+\infty} \frac{e^{xm} - e^{-xm}}{e^{xm} + e^{-xm}}.$$

333. Proposed by R. D. CARMICHAEL, Princeton University.

Sum the infinite series

$$\frac{1}{(m+1)^2} + \frac{(2m-1)}{(2m+1)^2} + \frac{(3m-1)^2}{(3m+1)^4} + \frac{(4m-1)^3}{(4m+1)^5} + \frac{(5m-1)^4}{(5m+1)^6} + \ldots$$

[No solution of this problem has been received.]

334. Proposed by G. B. M. ZERR. A. M., Ph. D., Philadelphia, Pa.

Sum the series, $2^n - n\,2^{n-2} + \frac{n(n-3)}{2!}2^{n-4} - \frac{n(n-4)(n-5)}{3!}2^{n-6}$

$$+ \frac{n(n-5)(n-6)(n-7)}{4!}2^{n-8} - \frac{n(n-6)(n-7)(n-8)(n-9)}{5!}2^{n-10} + \ldots$$

Solution by the PROPOSER.

We have, $(1-px)(1-qx)\equiv 1-x(p+q-pqx)$. Taking logarithms,

$$\log(1-px)+\log(1-qx)=\log[1-x(p+q-pqx)].$$

Hence, $(p+q)x+\dfrac{(p^2+q^2)x^2}{2}+\dfrac{(p^3+q^3)x^3}{3}+\ldots+\dfrac{(p^n+q^n)x^n}{n}$

$$=x(p+q-pqx)+\frac{x^2(p+q-pqx)^2}{2}+\ldots+\frac{x^n(p+q-pqx)^n}{n}.$$

The coefficients of x^n are equal for values of x, which makes the series convergent.

$$\therefore \frac{p^n+q^n}{n}=\frac{(p+q)^n}{n}-\frac{(n-1)pq(p+q)^{n-2}}{n-1}$$

$$+\frac{(n-2)(n-3)p^2q^2(p+q)^{n-4}}{2!(n-2)}-\frac{(n-3)(n-4)(n-5)p^3q^3(p+q)^{n-6}}{3!(n-3)}+\ldots$$

Hence, $p^n+q^n=(p+q)^n-npq(p+q)^{n-2}$

$$+\frac{n(n-3)}{2!}p^2q^2(p+q)^{n-4}-\frac{n(n-4)(n-5)}{3!}p^3q^3(p+q)^{n-6}+\ldots$$

Let $p=q$. Then $2=2^n-n2^{n-2}+\dfrac{n(n-3)}{2!}2^{n-4}-\dfrac{n(n-4)(n-5)}{3!}2^{n-6}$

$$+\frac{n(n-5)(n-6)(n-7)}{4!}2^{n-8}-\ldots$$

Solved similarly by V. M. Spunar, who starts with the identity, $2\log(1-x)=\log(1-2x+x^2)$.